Ref. SISSA 60/2002/EP
DO-TH 02/12
August 2002
hep-ph/0208XXX

On the CP Violation Associated with Majorana Neutrinos and Neutrinoless Double-Beta Decay

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Abstract

Assuming 3-ν mixing and massive Majorana neutrinos, we analyze the possibility of establishing the existence of CP-violation associated with Majorana neutrinos in the lepton sector by i) measuring of the effective Majorana mass $|<m>|$ in neutrinoless double beta decay with a sufficient precision and ii) by measuring of, or obtaining a stringent upper limit on, the lightest neutrino mass m_1. Information on m_1 can be obtained in the ^{3}H β-decay experiment KATRIN and from astrophysical and cosmological observations. Proving that the indicated CP-violation takes place requires, in particular, a relative experimental error on the measured value of $|<m>|$ not bigger than 20%, a "theoretical uncertainty" in the value of $|<m>|$ due to an imprecise knowledge of the corresponding nuclear matrix elements smaller than a factor of 2, a value of $\tan^2\theta_\odot \gtrsim 0.55$, and values of the relevant Majorana CP-violating phases typically within the intervals of $\sim (\pi/2 - 3\pi/4)$ and $\sim (5\pi/4 - 3\pi/2)$.

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1 Introduction

The recent results of the SNO solar neutrino experiment [1, 2] (see also [3]) provided further strong evidences for oscillations or transitions of the solar ν_e into active neutrinos $\nu_{\mu(\tau)}$ (and/or antineutrinos $\bar{\nu}_{\mu(\tau)}$). These evidences become even stronger when the SNO data are combined with the data obtained in the other solar neutrino experiments, Homestake, Kamiokande, SAGE, GALLEX/GNO and Super-Kamiokande [4, 5]. As the two-neutrino oscillation analyzes of the solar neutrino data show (see, e.g., [1]), the latter favor the large mixing angle (LMA) MSW $\nu_e \rightarrow \nu_{\mu(\tau)}$ transition solution with $\Delta m^2_\odot \sim 5 \times 10^{-5}$ eV^2 and $\tan^2\theta_\odot \sim 0.33$, $\tan^2\theta_\odot < 1$, where $\Delta m^2_\odot$ and $\theta_\odot$ are the neutrino mass squared difference and mixing angle which control the solar neutrino transitions. Strong evidences for oscillations of atmospheric neutrinos have been obtained in the Super-Kamiokande experiment [6]. The atmospheric neutrino data, as is well known, is best described in terms of dominant $\nu_\mu \rightarrow \nu_\tau$ ($\bar{\nu}_\mu \rightarrow \bar{\nu}_\tau$) oscillations with $|\Delta m^2_{\rm atm}| \sim (2.5 - 3.0) \times 10^{-3}$ eV^2.

The explanation of the solar and atmospheric neutrino data in terms of neutrino oscillations requires the existence of 3-neutrino mixing in the weak charged lepton current (see, e.g., [7, 8]):

$$\nu_{l\mathrm{L}} = \sum_{j=1}^{3} U_{lj}\, \nu_{j\mathrm{L}} \ . \tag{1}$$

Here ν_{lL}, $l = e, \mu, \tau$, are the three left-handed flavor neutrino fields, $\nu_{j\mathrm{L}}$ is the left-handed field of the neutrino ν_j having a mass m_j and U is the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) neutrino mixing matrix [9, 10]. If the neutrinos with definite mass ν_j are Majorana particles, the process of neutrinoless double-beta ($(\beta\beta)_{0\nu}$-) decay, $(A,Z) \rightarrow (A, Z+2) + e^- + e^-$, (A,Z) and $(A, Z+2)$ being initial and final state nuclei, will be allowed (for reviews see, e.g., [11, 12]). For Majorana neutrinos ν_j with masses not exceeding few MeV, the dependence of the $(\beta\beta)_{0\nu}$-decay amplitude on the neutrino mass and mixing parameters is confined to one factor — the effective Majorana mass $|<\!m\!>|$, which can be written in the form (see, e.g., [11]):

$$|<\!m\!>| = \left| m_1 |U_{\mathrm{e}1}|^2 + m_2 |U_{\mathrm{e}2}|^2 \; e^{i\alpha_{21}} + m_3 |U_{\mathrm{e}3}|^2 \; e^{i\alpha_{31}} \right| \tag{2}$$

where α_{21} and α_{31} are the two Majorana CP-violating phases [2] [13, 14]. If CP-invariance holds, one has [15, 16] $\alpha_{21} = k\pi$, $\alpha_{31} = k'\pi$, where $k, k' = 0, 1, 2, ...$ In this case

$$\eta_{21} \equiv e^{i\alpha_{21}} = \pm 1, \quad \eta_{31} \equiv e^{i\alpha_{31}} = \pm 1, \tag{3}$$

represent the relative CP-parities of the neutrinos ν_1 and ν_2, and ν_1 and ν_3, respectively.

One can express [17, 18, 19, 20, 21] the masses m_2 and m_3 entering into eq. (2) for $|<\!m\!>|$ in terms of $\Delta m^2_\odot$ and $\Delta m^2_{\rm atm}$, measured in the solar and atmospheric neutrino experiments, and m_1, while $|U_{\mathrm{e}j}|^2$, $j = 1, 2, 3$, are related to the mixing angle which controls the solar ν_e transitions $\theta_\odot$, and to the lepton mixing parameter $\sin^2\theta$ limited by the data from the CHOOZ and Palo Verde experiments [22, 23]. Within the convention $m_1 < m_2 < m_3$ we are going to use in what follows, one has $\Delta m^2_{\rm atm} \equiv \Delta m^2_{31}$, where $\Delta m^2_{jk} \equiv m^2_j - m^2_k$, and $m_3 = \sqrt{m^2_1 + \Delta m^2_{\rm atm}}$. For $\Delta m^2_\odot$ there are two possibilities, $\Delta m^2_\odot \equiv \Delta m^2_{21}$ and $\Delta m^2_\odot \equiv \Delta m^2_{32}$, corresponding respectively to two different types of neutrino mass spectrum — with normal and with inverted hierarchy. In the first case one has $m_2 = \sqrt{m^2_1 + \Delta m^2_\odot}$, $|U_{\mathrm{e}1}|^2 = \cos^2\theta_\odot(1 - |U_{\mathrm{e}3}|^2)$, $|U_{\mathrm{e}2}|^2 = \sin^2\theta_\odot(1 - |U_{\mathrm{e}3}|^2)$, and $|U_{\mathrm{e}3}|^2 \equiv \sin^2\theta$, while in the second $m_2 = \sqrt{m^2_1 + \Delta m^2_{\rm atm} - \Delta m^2_\odot}$, $|U_{\mathrm{e}2}|^2 = \cos^2\theta_\odot(1 - |U_{\mathrm{e}1}|^2)$, $|U_{\mathrm{e}3}|^2 = \sin^2\theta_\odot(1 - |U_{\mathrm{e}1}|^2)$, and $|U_{\mathrm{e}1}|^2 \equiv \sin^2\theta$. Thus, given $\Delta m^2_\odot$, $\Delta m^2_{\rm atm}$, $\theta_\odot$ and $\sin^2\theta$, $|<\!m\!>|$ depends, in general, on

[2] We assume that $m_j > 0$ and that the fields of the Majorana neutrinos ν_j satisfy the Majorana condition: $C(\bar{\nu}_j)^T = \nu_j$, $j = 1, 2, 3$, where C is the charge conjugation matrix.

the lightest neutrino mass m_1, on the two Majorana CP-violating phases α_{21} and α_{31} and on the "discrete ambiguity" related to the two possible types of neutrino mass spectrum. In the case of quasi-degenerate (QD) neutrino mass spectrum, $m_1 \cong m_2 \cong m_3$, $m_1^2 \gg \Delta m^2_{\rm atm}, \Delta m^2_{\odot}$, $|<m>|$ essentially does not depend on $\Delta m^2_{\rm atm}$ and $\Delta m^2_{\odot}$, and the two possibilities, $\Delta m^2_{\odot} \equiv \Delta m^2_{21}$ and $\Delta m^2_{\odot} \equiv \Delta m^2_{32}$, lead to the same predictions for [3] $|<m>|$.

The observation of $(\beta\beta)_{0\nu}$-decay will have fundamental implications for our understanding of the elementary particle interactions. It would imply, in particular, that the electron lepton charge L_e and the total lepton charge L are not conserved and can change by two units in the latter, and would suggest that the massive neutrinos are Majorana particles. Under the general and plausible assumptions of 3-ν mixing and massive Majorana neutrinos, $(\beta\beta)_{0\nu}$-decay generated only by the (V-A) charged current weak interaction via the exchange of the three Majorana neutrinos, and neutrino oscillation explanation of the solar and atmospheric neutrino data, which will be assumed to hold throughout this study, the observation of $(\beta\beta)_{0\nu}$-decay [4] can give unique information on the type of the neutrino mass spectrum and on the lightest neutrino mass, i.e., on the absolute scale of neutrino masses [19, 20, 21, 26, 27, 28, 29, 30, 31, 32]. One of the important implications of the latest results of the solar neutrino experiments, notably of SNO, which show that $\tan^2\theta_{\odot} < 1$ and, e.g., $\cos 2\theta_{\odot} \gtrsim 0.26$ at 99.73% C.L. [1], regard the predictions for $|<m>|$. The fact that $\cos 2\theta_{\odot}$ is significantly different from zero leads to [30] (see also [20, 28, 29]) the existence of significant lower bounds on $|<m>|$ (exceeding 0.01 eV) in the cases of neutrino mass spectrum with inverted hierarchy (IH) and of the quasi-degenerate (QD) type, and of a stringent upper bound (smaller than 0.01 eV) if the neutrino mass spectrum is with normal hierarchy (NH). Using, e.g., the best fit values of $\Delta m^2_{\odot}$, $\Delta m^2_{\rm atm}$, $\cos 2\theta_{\odot}$ and $\sin^2\theta$, obtained in the analyzes of the solar and atmospheric neutrino, and CHOOZ data in [1, 33], one finds respectively for the three types of spectra [30]: $|<m>| \gtrsim 2.8 \times 10^{-2}$ eV (IH), $|<m>| \gtrsim 0.06$ eV (QD) and $|<m>| \lesssim 2.0 \times 10^{-3}$ eV (NH). At 90% C.L. the indicated lower and upper bounds read, respectively: $|<m>| \gtrsim 1.5 \times 10^{-2}$ eV (IH), $|<m>| \gtrsim 0.25$ eV (QD) and $|<m>| \lesssim 6.0 \times 10^{-3}$ eV (NH). The quoted lower bounds are in the range of the sensitivity of currently operating and planned $(\beta\beta)_{0\nu}$-decay experiments (see further). These results imply, in particular, that a measured value of $|<m>| \neq 0$ (or an experimental upper limit on $|<m>|$) of the order of few $\times 10^{-2}$ eV can provide unique constraints on, or even can allow one to determine, the type of the neutrino mass spectrum in the case the massive neutrinos are Majorana particles; it can provide also a significant upper limit on the mass of the lightest neutrino m_1 [28, 29, 30]. Information on absolute values of neutrino masses in the range of interest can also be obtained in the ^{3}H β-decay neutrino mass experiment KATRIN [34] and from cosmological and astrophysical data (see, e.g., refs. [32, 35]).

Rather stringent upper bounds on $|<m>|$ have been obtained in the ^{76}Ge experiments by the Heidelberg-Moscow collaboration [36], $|<m>| < 0.35$ eV (90% C.L.), and by the IGEX collaboration [37], $|<m>| < (0.33 \div 1.35)$ eV (90% C.L.). Taking into account a factor of 3 uncertainty in the calculated value of the corresponding nuclear matrix element, we get for the upper limit found in [36]: $|<m>| < 1.05$ eV. Considerably higher sensitivity to the value of $|<m>|$ is planned to be reached in several $(\beta\beta)_{0\nu}$-decay experiments of a new generation. The NEMO3 experiment [38], which began to take data in July of 2002, and the cryogenics detector CUORICINO [39] to be operative in the second half of 2002, are expected to reach a sensitivity to values of $|<m>| \sim 0.2$ eV. Up to an order of magnitude better sensitivity, i.e., to $|<m>| \cong 2.7 \times 10^{-2}$ eV, 1.5×10^{-2} eV, 5.0×10^{-2} eV, 2.5×10^{-2} eV and 3.6×10^{-2} eV is planned to be achieved in the CUORE [39], GENIUS [40], EXO [41], MAJORANA [42] and MOON [43] experiments [5], respectively.

[3] This statement is valid as long as there are no independent constraints on the CP-violating phases α_{21} and α_{31} which enter into the expression for $|<m>|$.

[4] Evidences for $(\beta\beta)_{0\nu}$-decay taking place with a rate corresponding to 0.11 eV $\leq |<m>| \leq$ 0.56 eV (95% C.L.) are claimed to have been obtained in [24]. The results announced in [24] have been criticized in [25].

[5] The quoted sensitivities correspond to values of the relevant nuclear matrix elements taken from ref. [44].

In what regards the ^{3}H β-decay experiments, the currently existing most stringent upper bounds on the electron (anti-)neutrino mass $m_{\bar{\nu}_e}$ were obtained in the Troitzk [45] and Mainz [46] experiments and read $m_{\bar{\nu}_e} < 2.2$ eV. The KATRIN ^{3}H β-decay experiment [34] is planned to reach a sensitivity to $m_{\bar{\nu}_e} \sim 0.35$ eV.

In the present article we discuss the possibility of establishing the existence of CP-violation in the lepton sector due to the Majorana CP-violating phases by measuring $|<m>|$. The fundamental problem of CP-violation in the lepton sector is one of the most challenging future frontiers in the studies of neutrino mixing. It was noticed in [18] (see also [19]) that in the case of a *large mixing angle solution of the solar neutrino problem*, the observation of $(\beta\beta)_{0\nu}$-decay combined with data on the neutrino masses from ^{3}H β-decay experiments could provide, in principle, unique information on the CP-violation due to the Majorana CP-violating phases. As a more detailed study showed [20], information on the CP-violation of interest, and if CP-invariance holds — on the relative CP-parities of the massive Majorana neutrinos, could be obtained as well from a measurement of $|<m>|$ supplemented by information on the type of the neutrino mass spectrum (or the lightest neutrino mass m_1). The problem of interest was studied in detail for the QD neutrino mass spectrum (taking into account the relevant nuclear matrix element uncertainties) in [26]. Further analysis [6] showed [28] that a measurement of $|<m>|$ alone could exclude the possibility of both Majorana CP-violating phases α_{21} and α_{31} being equal to zero. However, such a measurement cannot rule out without additional input that the two phases take different CP-conserving values. The additional input needed for establishing CP-violation could be, e.g., the measurement of neutrino mass $m_{\bar{\nu}_e}$ in ^{3}H β-decay experiment KATRIN [34], or the cosmological determination of the sum of the three neutrino masses [35], $\Sigma = m_1 + m_2 + m_3$, or a derivation of a sufficiently stringent upper limit on m_1 (or Σ). It was also pointed out in [28] that the possibility of finding CP-violation "requires quite accurate measurements" of $|<m>|$ and, say, of $m_{\bar{\nu}_e}$, "and holds only for a limited range of values of the relevant parameters". The aim of the present paper is to quantify these requirements, and to better determine the ranges of values of the parameters in question, which could allow to detect CP-violation due to the Majorana CP-violating phases. We discuss also the requirement the possibility of establishing CP-violation imposes on the uncertainty in the values of the $(\beta\beta)_{0\nu}$-decay nuclear matrix elements. Let us add that at present no viable alternative to the measurement of $|<m>|$ for getting information about the Majorana CP-violating phases α_{21} and α_{31} exists (see, e.g., [49]), or can be foreseen to exist in the next ~ 8 years.

The problem of detecting CP-violation associated with Majorana neutrinos by measuring $|<m>|$ and $m_{\bar{\nu}_e}$ (or Σ) was discussed recently also in ref. [48]. The authors of [48], after making a certain number of assumptions about the experimental and theoretical developments in the field of interest that might occur by 2020 [7], claim to have shown "once and for all that it is impossible to detect CP-violation from $(\beta\beta)_{0\nu}$-decay in the foreseeable future." We have strong doubts that it is possible to foresee with certainty all the scientific and technological developments relevant to the problem of interest, which will take place in the next $\sim (10 - 18)$ years. Correspondingly, the approach we follow in the present work is "orthogonal" to that adopted in [48]: here we make an attempt to determine the conditions under which CP-violation might be detected from a measurement of $|<m>|$ and $m_{\bar{\nu}_e}$ (or Σ), or of $|<m>|$ and a sufficiently stringent upper limit on m_1 or Σ.

2 The Neutrino Mass and Oscillation Data and the Predictions for the Effective Majorana Mass $|<m>|$

As we have seen, the predicted value of $|<m>|$ depends in the case of 3-neutrino mixing of interest on: i) the value of the lightest neutrino mass m_1, ii) $\Delta m^2_{\odot}$ and $\theta_{\odot}$, iii) $\Delta m^2_{\rm atm}$, and iv) the lepton mixing angle θ which is limited by the CHOOZ and Palo Verde experiments [22, 23]. Given

[6]Aspects of the phenomenology of the effects of CP-violation in $(\beta\beta)_{0\nu}$-decay were discussed also, e.g., in [47].

[7]It is supposed in [48], in particular, that $|<m>|$ will be measured with a 25% (1 s.d.) error and that the uncertainty in the $(\beta\beta)_{0\nu}$-decay nuclear matrix elements will be reduced to a factor of 2.

the indicated parameters, the value of $|<m>|$ depends strongly on the type of the neutrino mass spectrum, as well as on the values of the two Majorana CP-violating phases, α_{21} and α_{31} (see eq. (2)), present in the lepton mixing matrix.

The possibility of detecting of CP-violation due to the Majorana CP-violating phases α_{21} and α_{31} if $|<m>|$ is found to be nonzero in the $(\beta\beta)_{0\nu}$-decay experiments of the next generation, depends crucially on the precision with which m_1 (or Σ), $\Delta m^2_\odot$, $\theta_\odot$, $\Delta m^2_{\rm atm}$, $\sin^2\theta$ and $|<m>|$ will be measured. It depends also crucially, as we shall see, on the values of m_1 or Σ, of $\theta_\odot$ and of $|<m>|$. Actually, the accuracy of measurement of $|<m>|$ in the next generation of $(\beta\beta)_{0\nu}$-decay experiments, given their sensitivity limits of $\sim (1.5-5.0)\times 10^{-2}$ eV, depends on the value of $|<m>|$. If only an upper limit on m_1 will be obtained, the possibility we are discussing will depend on how stringent this upper limit is. In what regards the dependence of $|<m>|$ on the type of the neutrino mass spectrum (normal versus inverted hierarchy), if the latter will not be determined in neutrino oscillation experiments (see, e.g., [50, 51]), the measurement of $|<m>| \neq 0$ itself in the next generation of $(\beta\beta)_{0\nu}$-decay experiments could provide this information through the value of $|<m>|$ found [30].

The value of m_1 can be measured by the KATRIN experiment [34] if the neutrino mass spectrum is of the QD type. In this case $m_1 \cong m_2 \cong m_3 \cong m_{\bar{\nu}_e}$. Given the currently allowed regions of values of $\Delta m^2_\odot$ and $\Delta m^2_{\rm atm}$ (see further), we have QD spectrum for $m_{1,2,3} \cong m_{\bar{\nu}_e} > 0.20$ eV. The KATRIN detector is designed to have a 1 s.d. error of 0.08 eV^2 on a measured value of $m^2_{\bar{\nu}_e}$. The most stringent upper limit on $m_{\bar{\nu}_e} \cong m_1$, which can be reached in this experiment, is 0.35 eV. The KATRIN experiment is expected to start in 2007.

The sum of neutrino masses Σ can be determined by using data on the weak lensing of galaxies by large scale structure, and data on the cosmic microwave background (CMB) from the MAP and PLANCK experiments, with an estimated (1 s.d.) error of 0.04 eV [35]. The latter represents the estimated best precision that can possibly be achieved in the cosmological determination of Σ. If only an upper limit of $\sim (0.10 - 0.15)$ eV on Σ will be obtained, that would strongly disfavor (if not rule out) the QD spectrum, while a measured value of $|<m>| \gtrsim 0.03$ eV would rule out the NH spectrum [30]. Given $\Delta m^2_\odot$ and $\Delta m^2_{\rm atm}$, the indicated upper limit on Σ could be used to derive a rather stringent upper limit on m_1: using $\Sigma < 0.15$ (0.12) eV and the current best fit values of $\Delta m^2_\odot \cong 5.0\times 10^{-5}$ eV^2 and $\Delta m^2_{\rm atm} \cong 3.0\times 10^{-3}$ eV^2, we get in the case of IH spectrum: $m_1 < 0.03$ (0.01) eV. An upper limit on m_1 of the order of $(0.010 - 0.025)$ eV might be of crucial importance for establishing CP-violation due to the Majorana CP-violating phases [20, 28, 29].

The analysis of the solar neutrino data [1, 2, 3, 4, 5], including the latest SNO results, in terms of the hypothesis of $\nu_e \to \nu_{\mu(\tau)}$ oscillations/transitions of the solar ν_e shows [1] (see also, e.g., [52, 53]) that the data favor the LMA MSW solution with $\Delta m^2_\odot > 0$ and $\tan^2\theta_\odot < 1$. The LOW solution of the solar neutrino problem with transitions into active neutrinos is only allowed at approximately 99.73% C.L. [1]; there do not exist other solutions at the indicated confidence level. In the case of the LMA solution, the range of values of $\Delta m^2_\odot$ found in [1] at 99.73% C.L. reads:

$$\text{LMA MSW}: \quad 2.2\times 10^{-5}\ \text{eV}^2 \lesssim \Delta m^2_\odot \lesssim 2.0\times 10^{-4}\ \text{eV}^2 \quad (99.73\%\ \text{C.L.}). \tag{4}$$

The best fit value of $\Delta m^2_\odot$ obtained in [1] is $(\Delta m^2_\odot)_{\rm BF} = 5.0\times 10^{-5}$ eV^2. The mixing angle $\theta_\odot$ was found in the case of the LMA solution to lie in an interval which at 99.73% C.L. is given by [1]

$$\text{LMA MSW}: \quad 0.26 \lesssim \cos 2\theta_\odot \lesssim 0.64 \quad (99.73\%\ \text{C.L.}). \tag{5}$$

The best fit value of $\cos 2\theta_\odot$ in the LMA solution region is given by $(\cos 2\theta_\odot)_{\rm BF} = 0.50$.

Similar results have been obtained, e.g., in [52, 53]; in particular, the minimal allowed values of $\cos 2\theta_\odot$ in the LMA solution region found in [1] and in [52] at 99.73% C.L. practically coincide. The minimal allowed value of $\cos 2\theta_\odot$ obtained in [53] at 99.73% C.L. is 0.10. The best fit values of $\cos 2\theta_\odot$ found in [1, 52, 53] coincide, while that of $\Delta m^2_\odot$ obtained in [53], $(\Delta m^2_\odot)_{\rm BF} \cong 5.5\times 10^{-5}$ eV^2 is only slightly larger than the value found in [1, 52].

In the two-neutrino $\nu_\mu \to \nu_\tau$ ($\bar{\nu}_\mu \to \bar{\nu}_\tau$) oscillation analysis of the atmospheric neutrino data performed in [6] the following best fit value of $\Delta m^2_{\rm atm}$ was obtained: $(\Delta m^2_{\rm atm})_{\rm BF} \cong 2.5 \times 10^{-3}$ eV^2. At 99.73% C.L. $\Delta m^2_{\rm atm}$ was found to lie in the interval: $(1.5 - 5.0) \times 10^{-3}$ eV^2.

A 3-ν oscillation analysis of the CHOOZ data showed [54], e.g., that for $\Delta m^2_\odot \lesssim 10^{-4}$ eV^2, the limits on $\sin^2\theta$ practically coincide with those derived in the 2-ν oscillation analysis in ref. [22]. Combined 3-ν oscillation analyzes of the solar neutrino, Super-Kamiokande atmospheric neutrino and CHOOZ data were performed in [33, 55] under the assumption of $\Delta m^2_\odot \ll \Delta m^2_{\rm atm}$ (see, e.g., [7, 8, 56]). For the best fit values of $\Delta m^2_{\rm atm}$ and $\sin^2\theta$ the authors of [33] and [55] obtained, $(\Delta m^2_{\rm atm})_{\rm BF} \cong 3.1 \times 10^{-3}$ eV^2, $(\sin^2\theta)_{\rm BF} \cong 0.005$, and $(\Delta m^2_{\rm atm})_{\rm BF} \cong 2.7 \times 10^{-3}$ eV^2, $(\sin^2\theta)_{\rm BF} \cong 0$, respectively. It was found in [55], in particular, that $\sin^2\theta < 0.05$ at 99.73% C.L.

If $\Delta m^2_\odot \cong (2.5 - 10.0) \times 10^{-5}$ eV^2, which is favored by the solar neutrino data, the KamLAND experiment taking data at present will be able to measure $\Delta m^2_\odot$ with an 1 s.d. error of $\sim (3-5)\%$ (see, e.g., refs. [57] and [55, 58] and the articles quoted therein). Combining the data from the solar neutrino experiments and from KamLAND would permit to determine $\tan^2\theta_\odot$ with a high precision as well: the estimated (1 s.d.) error on $\tan^2\theta_\odot$ is [8] $\sim 5\%$ [57].

Similarly, if $\Delta m^2_{\rm atm}$ lies in the interval $\Delta m^2_{\rm atm} \cong (2.0 - 5.0) \times 10^{-3}$ eV^2, as is suggested by the current atmospheric neutrino data [6], its value will be determined with a $\sim 10\%$ error (1 s.d.) by the MINOS experiment [59] which is scheduled to start in December of 2004. Somewhat better limits on $\sin^2\theta$ than the existing one can be obtained in the MINOS experiment [59] as well. Various options are being currently discussed (experiments with off-axis neutrino beams, more precise reactor antineutrino and long base-line experiments, etc., see, e.g., [61]) of how to improve by at least an order of magnitude, i.e., to values of ~ 0.005 or smaller, the sensitivity to $\sin^2\theta$.

All the indicated developments are expected to take place within the next $\sim (7-8)$ years, i.e., by 2010. We will assume in what follows that the problem of measuring or tightly constraining $\sin^2\theta$ will also be resolved within the indicated period. We will also assume that by 2010 one or more $(\beta\beta)_{0\nu}$-decay experiments of the next generation will be operative, and that at least the physical range of variation of the values of the relevant $(\beta\beta)_{0\nu}$-decay nuclear matrix elements will be unambiguously determined. Since all discussed future experiments (except the 10 ton version of GENIUS [40]) have a sensitivity to $|<\!m\!>| > 0.01$ eV, while in the case of NH neutrino mass spectrum $|<\!m\!>|$ is predicted to be smaller than 0.01 eV, we will not consider the possibility of finding CP-violation for the NH spectrum (for a discussion of this possibility see, e.g., [28, 29]).

We begin with a general analysis the aim of which is to determine what is the maximal uncertainty in the value of $|<\!m\!>|$ due to the imprecise knowledge of the corresponding nuclear matrix elements, which might still allow one to find CP-violation associated with Majorana neutrinos. This is followed by results on the problem of finding the CP-violation of interest, derived by a simplified error analysis of prospective input data on $m_{\bar{\nu}_e}$ or Σ, $\tan^2\theta_\odot$, $|<\!m\!>|$, etc. The effects of the nuclear matrix element uncertainty on the results of the statistical analysis are also considered.

3 Finding CP-Violation Associated with Majorana Neutrinos

3.1 General Constraints on the Nuclear Matrix Element Uncertainty

3.1.1 Inverted Neutrino Mass Hierarchy: $\Delta m^2_\odot \equiv \Delta m^2_{32}$, $m_1 < 0.02$ eV

If $\Delta m^2_\odot = \Delta m^2_{32}$, one has [19, 20]:

$$
\begin{aligned}
|<\!m\!>| \; = \; & \Big| m_1 |U_{\rm e1}|^2 + \sqrt{m_1^2 + \Delta m^2_{\rm atm} - \Delta m^2_\odot}\, \cos^2\theta_\odot (1 - |U_{\rm e1}|^2) e^{i\alpha_{21}} \\
& + \sqrt{m_1^2 + \Delta m^2_{\rm atm}}\, \sin^2\theta_\odot (1 - |U_{\rm e1}|^2) e^{i\alpha_{31}} \Big|
\end{aligned} \tag{6}
$$

$$
\simeq \left| \sqrt{\Delta m^2_{\rm atm}} \left(\cos^2\theta_\odot + \sin^2\theta_\odot e^{i\alpha_{32}} \right) \right| = \sqrt{\Delta m^2_{\rm atm}} \sqrt{1 - \sin^2 2\theta_\odot \; \sin^2 \tfrac{\alpha_{32}}{2}} \; , \tag{7}
$$

[8]We thank C. Peña-Garay for clarifications on this point.

where $\alpha_{32} = \alpha_{31} - \alpha_{21}$. In eq. (7) we have neglected $\Delta m^2_{\odot}$ and m_1^2 with respect to $\Delta m^2_{\rm atm}$ as well as the terms proportional to $|U_{\rm e1}|^2$ which is limited by the CHOOZ data. For the best fit value of $\Delta m^2_{\rm atm} \cong 3.0 \times 10^{-3}$ eV^2, $m_1 < 0.01$ eV and the best fit value of $\Delta m^2_{\odot} \cong (5.0 - 5.5) \times 10^{-5}$ eV^2, the corrections due to the $\Delta m^2_{\odot}$ and m_1^2 do not exceed 1%. The same conclusion is valid for the corrections due to the terms $\sim |U_{\rm e1}|^2$ as long as $|U_{\rm e1}|^2 \lesssim 0.01$. For, e.g, $\Delta m^2_{\odot} \cong 2 \times 10^{-4}$ eV^2, $|U_{\rm e1}|^2 \cong 0.04$ and $m_1 \cong 0.02$ eV, the corrections due to the terms neglected in eq. (7) can reach ~(5-6)%. These terms, obviously, should be taken into account if they turn out to have the indicated (or larger) values and $|\langle m \rangle|$ is measured with a comparable to the corrections, or somewhat larger, experimental error. In the latter case it will be necessary to use the exact formula for $|\langle m \rangle|$, eq. (6), in order not to introduce avoidable sources of uncertainties. We are interested here only i) in analyzing the impact for the searches of CP-violation associated with the Majorana CP-violating phases of the uncertainty in $|\langle m \rangle|$, caused by the uncertainty in the evaluation of the relevant $(\beta\beta)_{0\nu}$-decay nuclear matrix elements, and ii) in deriving limits on the nuclear matrix element uncertainty, which, if satisfied, could permit one to draw conclusions concerning the CP-violation of interest. Therefore in what follows we will use the approximate expression for $|\langle m \rangle|$, eq. (7), but our results can be easily generalized using the exact formula, eq. (6).

A positive signal for $(\beta\beta)_{0\nu}$-decay in the future experiments with $\sqrt{(\Delta m^2_{\rm atm})_{\rm MIN}}\,(\cos 2\theta_{\odot})_{\rm MIN} \leq |\langle m \rangle| \leq \sqrt{(\Delta m^2_{\rm atm})_{\rm MAX}}$, where $(\Delta m^2_{\rm atm})_{\rm MIN}$, $(\Delta m^2_{\rm atm})_{\rm MAX}$ and $(\cos 2\theta_{\odot})_{\rm MIN}$ are determined from the experimentally measured values taking a given C.L. interval, combined with an upper bound on m_1, $m_1 < 0.02$ eV, would lead to the conclusion that the neutrino mass spectrum is of the IH type. A "just-CP-violating" region [20] — a value of $|\langle m \rangle|$ in this region would signal unambiguously CP-violation in the lepton sector due to Majorana CP-violating phases, would be present if

$$(|\langle m \rangle|_{\rm exp})_{\rm MAX} < \sqrt{(\Delta m^2_{\rm atm})_{\rm MIN}} \tag{8}$$

$$(|\langle m \rangle|_{\rm exp})_{\rm MIN} > \sqrt{(\Delta m^2_{\rm atm})_{\rm MAX}}\,(\cos 2\theta_{\odot})_{\rm MAX}, \tag{9}$$

where $(|\langle m \rangle|_{\rm exp})_{\rm MAX(MIN)}$ is the largest (smallest) experimentally allowed value of $|\langle m \rangle|$, taking into account both the experimental error on the measured $(\beta\beta)_{0\nu}$-decay half life-time and the uncertainty due to the evaluation of the nuclear matrix elements. Condition (9) depends crucially on the value of $(\cos 2\theta_{\odot})_{\rm MAX}$ and it is less stringent for smaller values of $(\cos 2\theta_{\odot})_{\rm MAX}$ [28].

We can parametrize the uncertainty in $|\langle m \rangle|$ due to the poor knowledge of the relevant nuclear matrix elements — we will use the term "theoretical uncertainty" for the latter, through a parameter ζ, $\zeta \geq 1$, defined as:

$$|\langle m \rangle| = \zeta \Big((|\langle m \rangle|_{\rm exp})_{\rm MIN} \pm \Delta \Big) \tag{10}$$

where $(|\langle m \rangle|_{\rm exp})_{\rm MIN}$ is the value of $|\langle m \rangle|$ obtained from the measured $(\beta\beta)_{0\nu}$-decay half life-time of a given nucleus using *the largest nuclear matrix element* and Δ is the experimental error. The necessary condition permitting to establish, in principle, that the CP-symmetry is violated due to the Majorana CP-violating phases is:

$$1 \leq \zeta < \frac{\sqrt{(\Delta m^2_{\rm atm})_{\rm MIN}}}{\sqrt{(\Delta m^2_{\rm atm})_{\rm MAX}}\,(\cos 2\theta_{\odot})_{\rm MAX} + 2\Delta}\,. \tag{11}$$

Obviously, the smaller $(\cos 2\theta_{\odot})_{\rm MAX}$ and Δ the larger the "theoretical uncertainty" which might allow one to make conclusions concerning the CP-violation of interest.

From here on, for the sake of simplicity, we assume that the "theoretical uncertainty" dominates over the experimental one and we neglect Δ. If the computation of the nuclear matrix elements becomes sufficiently accurate and/or if Δ is relatively large, it would be necessary to take into account also the uncertainty due to the experimental error in the analysis which follows.

For the best fit value of $\Delta m^2_{\rm atm}$ [33], $(\Delta m^2_{\rm atm})_{\rm BF} = 3.1 \times 10^{-3}$ eV, taking $(\cos 2\theta_\odot)_{\rm BF} = 0.5, 0.4, 0.3$ and allowing for 10% (20%) uncertainty in the values of both parameters [9], condition (11) (with negligible Δ) implies $\zeta < 1.65, 2.05, 2.73$ (1.33, 1.67, 2.22), respectively.

Condition (11) is obtained for the most favorable case in which the minimal allowed value of $|<m>|$, $(|<m>|_{\rm exp})_{\rm MIN}$, is very close to the upper bound of the allowed range of values for $|<m>|$ in the CP-conserving case and opposite CP-parities for the two relevant neutrinos, e.g., for $(|<m>|_{\rm exp})_{\rm MIN} = \sqrt{(\Delta m^2_{\rm atm})_{\rm MAX}}\,(\cos 2\theta_\odot)_{\rm MAX}$. In the general case this condition might not be satisfied. Let us parametrize the experimental value of $|<m>|$, $(|<m>|_{\rm exp})_{\rm MIN}$, obtained using *the largest nuclear matrix element*, as follows:

$$(|<m>|_{\rm exp})_{\rm MIN} = y\sqrt{(\Delta m^2_{\rm atm})_{\rm MIN}}\,(\cos 2\theta_\odot)_{\rm MIN}, \qquad y \geq 1. \tag{12}$$

Using eq. (10) we get:

$$|<m>| = \zeta y\sqrt{(\Delta m^2_{\rm atm})_{\rm MIN}}\,(\cos 2\theta_\odot)_{\rm MIN}. \tag{13}$$

The requirement that $|<m>|$ takes values in the region allowed in the case of neutrino mass spectrum with inverted hierarchy, translates into an interval of allowed values of y:

$$1 \leq y \leq \frac{\sqrt{(\Delta m^2_{\rm atm})_{\rm MAX}}}{\sqrt{(\Delta m^2_{\rm atm})_{\rm MIN}}}\frac{1}{(\cos 2\theta_\odot)_{\rm MIN}}. \tag{14}$$

The necessary conditions for establishing CP-violation due to the Majorana CP-violating phases, eqs. (8) and (9), lead to the following constraints on the parameters y and ζ:

$$\frac{\sqrt{(\Delta m^2_{\rm atm})_{\rm MAX}}}{\sqrt{(\Delta m^2_{\rm atm})_{\rm MIN}}}\frac{(\cos 2\theta_\odot)_{\rm MAX}}{(\cos 2\theta_\odot)_{\rm MIN}} < y < \frac{1}{(\cos 2\theta_\odot)_{\rm MIN}} \tag{15}$$

$$1 \leq \zeta < \frac{1}{y(\cos 2\theta_\odot)_{\rm MIN}}. \tag{16}$$

The necessary condition for CP- violation (11) (for negligible Δ) can be obtained from eq. (16) by taking $y = \sqrt{(\Delta m^2_{\rm atm})_{\rm MAX}}\,(\cos 2\theta_\odot)_{\rm MAX}/(\sqrt{(\Delta m^2_{\rm atm})_{\rm MIN}}\,(\cos 2\theta_\odot)_{\rm MIN})$.

In order to exclude the case of CP-conservation and equal CP-parities of the two relevant neutrinos ν_2 and ν_3, the following relation has to be satisfied:

$$(|<m>|)_{\rm MAX} < \sqrt{(\Delta m^2_{\rm atm})_{\rm MIN}}\,, \tag{17}$$

or, in terms of the parameters y and ζ,

$$\zeta y < \frac{1}{(\cos 2\theta_\odot)_{\rm MIN}}. \tag{18}$$

For $(\cos 2\theta_\odot)_{\rm MIN} = 0.40$ (0.30) and $y = 1.0,\ 1.5,\ 2.0$, ζ needs to satisfy $\zeta < 2.2,\ 1.8,\ 1.2$ (3.3, 2.2, 1.67).

The case of CP-conservation with ν_2 and ν_3 having opposite CP-parities, $\eta_{21} = -\eta_{31} = \pm 1$, can be excluded if

$$(|<m>|)_{\rm MIN} > \sqrt{(\Delta m^2_{\rm atm})_{\rm MAX}}\,(\cos 2\theta_\odot)_{\rm MAX}, \tag{19}$$

or, equivalently

$$y > \frac{\sqrt{(\Delta m^2_{\rm atm})_{\rm MAX}}\,(\cos 2\theta_\odot)_{\rm MAX}}{\sqrt{(\Delta m^2_{\rm atm})_{\rm MIN}}\,(\cos 2\theta_\odot)_{\rm MIN}}. \tag{20}$$

[9]In order for Δ to be negligible in eq. (11) for $\Delta m^2_{\rm atm} = 3.1 \times 10^{-3}$ eV, $\cos 2\theta_\odot = 0.4$ and an experimental error of 20% on both these parameters, one must have $\Delta \ll 0.01$ eV. Such a precision in the measurement of $|<m>|$ cannot be achieved in the planned next generation $(\beta\beta)_{0\nu}$-decay experiments, except possibly in the 10 ton version of GENIUS. Sufficiently small values of Δ can be achieved in practically all $(\beta\beta)_{0\nu}$-decay experiments of the next generation if $|<m>| \gtrsim 0.20$ eV.

For an uncertainty of 10% in the values of $\Delta m^2_{\rm atm}$ and $\cos 2\theta_\odot$, this inequality reads $y > 1.34$.

If the neutrino mass spectrum is of the inverted hierarchy type, a sufficiently precise determination of $\Delta m^2_{\rm atm}$, $\theta_\odot$ and $|U_{\rm e1}|^2$ (or a better upper limit on $|U_{\rm e1}|^2$), combined with a measurement of $|{<}m{>}|$ in the $(\beta\beta)_{0\nu}$-decay experiments, could allow one to get information on the difference of the Majorana CP-violating phases $(\alpha_{31} - \alpha_{21})$ [18]. The value of $\sin^2(\alpha_{31} - \alpha_{21})/2$ is related to the experimentally measurable quantities as follows [18, 19, 20]:

$$\sin^2 \frac{\alpha_{31} - \alpha_{21}}{2} \simeq \left(1 - \frac{|{<}m{>}|^2}{(\Delta m^2_{\rm atm} + m_1^2)(1 - |U_{\rm e1}|^2)^2}\right) \frac{1}{\sin^2 2\theta_\odot} \simeq \left(1 - \frac{|{<}m{>}|^2}{\Delta m^2_{\rm atm}}\right) \frac{1}{\sin^2 2\theta_\odot}, \tag{21}$$

where in writing the second simplified expression we have assumed that $|U_{\rm e1}|^2 \lesssim 0.01$ and $m_1 \lesssim 0.01$ eV. The constraints on $\sin^2(\alpha_{31} - \alpha_{21})/2 \neq 0, 1$, which correspond to CP-violation, are equivalent to eqs. (8) and (9). Given the fact that the atmospheric neutrino data implies $\sqrt{\Delta m^2_{\rm atm}} \gtrsim 0.04$ eV, obtaining, e.g., an experimental upper limit on $|{<}m{>}|$ of the order of 0.03 eV would permit, in particular, to get [30] a lower bound on the value of $\sin^2(\alpha_{31} - \alpha_{21})/2$ and possibly exclude the CP-conserving case corresponding to $\alpha_{31} - \alpha_{21} = 0$ (i.e., $\eta_{21} = \eta_{31} = \pm 1$). Note that one of the two CP-violating phases, α_{21} or α_{31}, will not be constrained in the case under discussion. Thus, even if it is found that $\alpha_{31} - \alpha_{21} = 0, \pm\pi$, at least one of the phases α_{21} and α_{31} can be a source of CP-violation in $\Delta L = 2$ processes other than $(\beta\beta)_{0\nu}$-decay. Let us note also that in the limit of negligible (zero) $|U_{\rm e1}|^2$ and m_1, there is practically only one physical CP-violating phase in the lepton sector in the case under discussion - the Majorana CP-violating phase $\alpha_{32} = \alpha_{31} - \alpha_{21}$.

3.1.2 Quasi-Degenerate Mass Spectrum ($m_1 > 0.2$ eV, $m_1 \simeq m_2 \simeq m_3 \simeq m_{\bar\nu_e}$)

For the QD neutrino mass spectrum, the effective Majorana mass $|{<}m{>}|$ is given in terms of $m_{\bar\nu_e} \cong m_{1,2,3}$, $\theta_\odot$ and $|U_{\rm e3}|^2$ which is constrained by the CHOOZ data, as follows (see, e.g., [20, 26]):

$$\begin{aligned} |{<}m{>}| &= m_{\bar\nu_e} \left| \cos^2\theta_\odot (1 - |U_{\rm e3}|^2) + \sin^2\theta_\odot (1 - |U_{\rm e3}|^2) e^{i\alpha_{21}} + |U_{\rm e3}|^2 e^{i\alpha_{31}} \right| && (22) \\ &\simeq m_{\bar\nu_e} \left| \cos^2\theta_\odot + \sin^2\theta_\odot e^{i\alpha_{21}} \right| = m_{\bar\nu_e} \sqrt{1 - \sin^2 2\theta_\odot \; \sin^2 \tfrac{\alpha_{21}}{2}}\ . && (23) \end{aligned}$$

In eq. (23) we have neglected $\Delta m^2_\odot$ and $\Delta m^2_{\rm atm}$ since in the case under consideration $m_1^2 \gg \Delta m^2_{\rm atm} \gg \Delta m^2_\odot$. We have furthermore neglected $|U_{\rm e3}|^2$, which leads to an uncertainty on $|{<}m{>}|$ not exceeding 2% if $|U_{\rm e3}|^2 < 0.01$. If $|U_{\rm e3}|^2$ turns out to have a value, e.g, close to its current upper limit, $|U_{\rm e3}|^2 \cong 0.04$, the correction due to $|U_{\rm e3}|^2$ in $|{<}m{>}|$ can be as large as $\mathcal{O}(8\% \div 12\%)$ and $|U_{\rm e3}|^2$ should be kept in the expression for $|{<}m{>}|$.

The effective Majorana mass $|{<}m{>}|$ in the case of QD spectrum is limited from below since $\cos 2\theta_\odot > 0$ and the inequality $m_1^2 \gg \Delta m^2_{\rm atm}$ implies $m_1 \cong m_{\bar\nu_e} > 0.2$ eV. The lower limit on $|{<}m{>}|$ is reached in the case of CP-conservation and $\eta_{21} = \eta_{31} = -1$. Using the best fit, the 90% C.L. and the 99.73% C.L., allowed values, of $\cos 2\theta_\odot$ from [1], we obtain [30], respectively, $|{<}m{>}| \gtrsim 0.10$ eV, $|{<}m{>}| \gtrsim 0.06$ eV and $|{<}m{>}| \gtrsim 0.035$ eV.

The indicated values of $|{<}m{>}|$ are in the range of sensitivity of some current (NEMO3, CUORICINO) and of most future $(\beta\beta)_{0\nu}$-decay experiments of the next generation. A measurement of $|{<}m{>}|$, $|{<}m{>}| > 0.10$ eV, would allow one to conclude that the neutrino mass spectrum is of the QD type. If, however, $|{<}m{>}|$ is found to lie in the interval 0.035 eV $\leq |{<}m{>}| \leq$ 0.10 eV, one would need additional information to establish that the neutrino masses are quasi-degenerate. For instance, the inequality $m_1^2 \cong m_{\bar\nu_e}^2 \gg \Delta m^2_{\rm atm}$, which for the current best fit value of $\Delta m^2_{\rm atm}$ corresponds to $m_1, m_{\bar\nu_e} > 0.2$ eV, should be fulfilled.

For the values of $|{<}m{>}|$ in the range of sensitivity of the discussed current and future $(\beta\beta)_{0\nu}$-decay experiments, a "just-CP-violation" region can exist. In order to establish whether CP-violation due to the Majorana CP-violating phases takes place, the uncertainty on the measured value of $m_{\bar{\nu}_e}$ should be sufficiently small. More specifically, the maximal value of $|<\!m\!>|$ for the allowed range of values of $m_{\bar{\nu}_e}$ in the case of CP-conservation and $\eta_{21} = -\eta_{31} = -1$ (see eq. (22)) must be smaller than the minimal value of $|<\!m\!>|$ for the same range of $m_{\bar{\nu}_e}$ and $\eta_{21} = -\eta_{31} = 1$. This leads to the following constraint on the allowed range of (or twice the relative error on) $m_{\bar{\nu}_e}$:

$$\frac{(m_{\bar{\nu}_e})_{\mathrm{MAX}} - (m_{\bar{\nu}_e})_{\mathrm{MIN}}}{(m_{\bar{\nu}_e})_{\mathrm{MAX}}} < 1 - (1 + |U_{\mathrm{e3}}|^2_{\mathrm{MAX}})(\cos 2\theta_\odot)_{\mathrm{MAX}} - |U_{\mathrm{e3}}|^2_{\mathrm{MAX}} \ , \tag{24}$$

where we have neglected the terms $\sim \mathcal{O}(|U_{\mathrm{e3}}|^2_{\mathrm{MAX}})^2$. Obviously, condition (24) is less constraining for smaller values of $(\cos 2\theta)_{\mathrm{MAX}}$.

If condition (24) is satisfied, the "just-CP-violation" interval of values of $|<\!m\!>|$ is given by:

$$(|<\!m\!>|_{\mathrm{exp}})_{\mathrm{MIN}} > \big((\cos 2\theta_\odot)_{\mathrm{MAX}}(1 - |U_{\mathrm{e3}}|^2_{\mathrm{MAX}}) + |U_{\mathrm{e3}}|^2_{\mathrm{MAX}}\big)(m_{\bar{\nu}_e})_{\mathrm{MAX}} \ , \tag{25}$$

$$(|<\!m\!>|_{\mathrm{exp}})_{\mathrm{MAX}} < \big(1 - 2|U_{\mathrm{e3}}|^2_{\mathrm{MAX}}\big)(m_{\bar{\nu}_e})_{\mathrm{MIN}} . \tag{26}$$

The necessary condition for CP-violation, eq. (25), is strongly dependent on $(\cos 2\theta_\odot)_{\mathrm{MAX}}$: the smaller the value of $(\cos 2\theta_\odot)_{\mathrm{MAX}}$, the larger the "just-CP-violation" region [28].

Assuming that $|U_{\mathrm{e3}}|^2 \lesssim 0.01$, we can neglect $|U_{\mathrm{e3}}|^2$ in eqs. (25) and (26). Similarly to the case of IH neutrino mass spectrum, one can parametrize the uncertainty in the value of $|<\!m\!>|$ associated with the theoretical uncertainty in the value of the relevant nuclear matrix element(s) by introducing two real parameters, ζ and y: ζ is determined by eq. (10), while the definition of y in the case under study can formally be obtained from eq. (13) by replacing $\sqrt{\Delta m^2_{\mathrm{atm}}}$ with $m_{\bar{\nu}_e}$. The parametrization of $|<\!m\!>|$, the necessary conditions for CP-violation due to Majorana CP-violating phases, the constraints on ζ and y, etc., can formally be obtained from those derived for the case of IH neutrino mass spectrum in Subsection 3.1.1 by substituting $\sqrt{\Delta m^2_{\mathrm{atm}}}$ with $m_{\bar{\nu}_e}$, and we are not going to give them here. Let us note only that, in general, the uncertainty in the measured value of $m_{\bar{\nu}_e}$ (or Σ) is expected to be larger than that in $\Delta m^2_{\mathrm{atm}}$ of $\mathcal{O}(10\%)$ we have assumed. Correspondingly, the former plays a more important role as limiting factor for the possibility of detecting the CP-violation under discussion (see Section 3.2).

A rather precise determination of $|<\!m\!>|$, $m_1 \cong m_{\bar{\nu}_e}$, $\theta_\odot$ and $|U_{\mathrm{e3}}|^2$ would imply an interdependent constraint on the two CP-violating phases α_{21} and α_{31} [20, 26] (see Fig. 16 in [20]). For $m_1 \equiv m_{\bar{\nu}_e} > 0.2$ eV, the phase α_{21} could be tightly constrained if $|U_{\mathrm{e3}}|^2$ is sufficiently small and the term in $|<\!m\!>|$ containing it can be neglected, as is suggested by the current limits on $|U_{\mathrm{e3}}|^2$:

$$\sin^2 \frac{\alpha_{21}}{2} \simeq \Big(1 - \frac{|<\!m\!>|^2}{m^2_{\bar{\nu}_e}}\Big)\frac{1}{\sin^2 2\theta_\odot}. \tag{27}$$

The term which depends on the CP-violating phase α_{31} in the expression for $|<\!m\!>|$, is suppressed by the factor $|U_{\mathrm{e3}}|^2$. Therefore the constraint one could possibly obtain on $\cos\alpha_{31}$ is trivial, unless $|U_{\mathrm{e3}}|^2 \sim \mathcal{O}(\sin^2\theta_\odot)$. The constraints $\sin^2(\alpha_{21}/2) \neq 0, 1$ implying CP-violation, are satisfied if the necessary conditions for CP-violation, eqs. (25) and (26), hold.

If η_{21} and η_{31} take the CP-conserving values $\eta_{21} = \pm\eta_{31} = -1$, there are both an upper and a lower bounds on $|<\!m\!>|$, $m_{\bar{\nu}_e}((\cos 2\theta_\odot)_{\mathrm{MIN}}(1 - |U_{\mathrm{e3}}|^2_{\mathrm{MIN}}) + |U_{\mathrm{e3}}|^2_{\mathrm{MIN}}) \leq |<\!m\!>| \leq m_{\bar{\nu}_e}((\cos 2\theta_\odot)_{\mathrm{MAX}}(1 - |U_{\mathrm{e3}}|^2_{\mathrm{MAX}}) + |U_{\mathrm{e3}}|^2_{\mathrm{MAX}})$, where we have used eq. (22). Given the range of allowed values of $\cos 2\theta_\odot$, the observation of the $(\beta\beta)_{0\nu}$-decay in the present and/or future $(\beta\beta)_{0\nu}$-decay experiments, combined with a sufficiently stringent upper limit on $m_{\bar{\nu}_e} \simeq m_{1,2,3}$, $m_{\bar{\nu}_e} < |<\!m\!>|_{\mathrm{exp}}/((\cos 2\theta_\odot)_{\mathrm{MAX}}(1 - |U_{\mathrm{e3}}|^2_{\mathrm{MAX}}) + |U_{\mathrm{e3}}|^2_{\mathrm{MAX}})$, would permit, e.g., to exclude the case of CP-conservation with $\eta_{21} = \pm\eta_{31} = -1$ [30].

3.2 Example of Simplified Error Analysis

In this subsection we give an example of simplified error analysis of prospective data on $m_{\bar{\nu}_e}$ or Σ, $\tan^2\theta_\odot$, $|<\!m\!>|$, etc., performed with the aim of establishing at a given C.L. that the phases

α_{21} and/or α_{31}, or $\alpha_{32} = \alpha_{31} - \alpha_{21}$, take CP-violating values. For simplicity we use eq. (23) (eq. (7)), assuming that $\sin^2\theta \equiv |U_{e3}|^2$ ($|U_{e1}|^2$) is sufficiently small, $\sin^2\theta \lesssim 0.01$. In this case we get for the CP-violating phase of interest either eq. (27) (QD spectrum) or eq. (21) (IH spectrum).

In the analysis which follows we use the following 4 representative values of $\tan^2\theta_\odot$ from the region of the LMA solution: $\tan^2\theta_\odot = 0.25;\ 0.40;\ 0.55;\ 0.70$. The 1 s.d. error of the experimentally measured value of $\tan^2\theta_\odot$ is assumed to be $\sigma(\tan^2\theta_\odot)/\tan^2\theta_\odot = 5\%$. Considering the case of QD neutrino mass spectrum, we take the following illustrative values of $m_0^2 \equiv m_{\bar{\nu}_e}^2 \cong m_{1,2,3}^2$ (to be measured in the ^{3}H β-decay experiment KATRIN) and Σ (which could be determined from cosmological and astrophysical data): $m_0^2 = (1.0)^2;\ (0.70)^2;\ (0.50)^2$ eV^2 and $\Sigma = 3.0;\ 1.5;\ 0.60$ eV, with $m_0 = \Sigma/3$. The error on the value of m_0^2 used in the analysis is $\sigma(m_0^2) = 0.08$ eV^2 [34], while that on the value of Σ is assumed to be $\sigma(\Sigma) = 0.04$ eV [35]. The latter implies for the QD spectrum that $\sigma(m_0) = \sigma(\Sigma)/3 \cong 0.013$ eV. We represent the error of the experimentally measured value of $|<m>|$ in the standard form:

$$\frac{\sigma(|<m>|)}{|<m>|} = \sqrt{(E_1)^2 + (E_2)^2}\ , \tag{28}$$

where E_1 and E_2 are the statistical and systematic errors. We choose $E_2 = const = 0.05$. We take $E_1 = f/|<m>|$, where we assume $f = 0.028$ eV. This gives a total relative error $\sigma(|<m>|)/|<m>| \cong 15\%$ at $|<m>| = 0.20$ eV. The above choices are motivated by the fact that the sensitivities of the next generation of $(\beta\beta)_{0\nu}$-decay experiments (CUORE, GENIUS, EXO, MAJORANA, MOON) in the measurement of $|<m>|$ are estimated to be in the range of $\sim (1.5 - 5.0) \times 10^{-2}$ eV and if, e.g, $|<m>| \gtrsim 0.20$ eV, a precision in the determination of $|<m>|$ corresponding to an error of $\sim 15\%$ could be reached in these experiments. Moreover, for values of $|<m>|$ which are sufficiently bigger than the quoted sensitivity limits of the future experiments, the statistical error scales as $|<m>|$ increases like $E_1 \sim const/|<m>|$.

The measurement of $|<m>|$ and m_0^2 and the more accurate determination of $\tan^2\theta_\odot$ would allow one to determine $\sin^2\alpha$, where $\alpha \equiv \alpha_{21}/2$ for the QD case, using eq. (27). Using error multiplication, the error on $\sin^2\alpha$ is:

$$\sigma(\sin^2\alpha) = \frac{(1+\tan^2\theta_\odot)^2}{4\ \tan^2\theta_\odot}\left[4\left(\frac{|<m>|^2}{m_0^2}\right)^2\left(\frac{\sigma(|<m>|)}{|<m>|}\right)^2 + \left(\frac{|<m>|^2}{m_0^2}\right)^2\left(\frac{\sigma(m_0^2)}{m_0^2}\right)^2\right.$$
$$\left. + \left(\frac{1-\tan^2\theta_\odot}{1+\tan^2\theta_\odot}\right)^2\left(1 - \frac{|<m>|^2}{m_0^2}\right)^2\left(\frac{\sigma(\tan^2\theta_\odot)}{\tan^2\theta_\odot}\right)^2\right]^{1/2} . \tag{29}$$

If the sum $\Sigma = 3m_0$ is cosmologically determined, one has $\sigma(\Sigma)/\Sigma = \sigma(m_0)/m_0$ and the error on $\sin^2\alpha$ can be obtained from the above equation by using $\sigma(m_0^2)/m_0^2 = 2\sigma(m_0)/m_0$.

In deriving eq. (29) we have assumed that there are no correlations between the errors on m_0^2 (or m_0), $\tan^2\theta_\odot$ and $|<m>|$. Let us note that the first two terms in the expression for $\sigma(\sin^2\alpha)$ are suppressed for $m_0^2 \gg |<m>|^2$, i.e., for $\alpha \sim \pi/2$. The third is small for $|<m>| \simeq m_0$, i.e., for $\alpha \sim 0, \pi$, when the first two dominate, and/or for $\tan^2\theta_\odot \simeq 1$. The common factor in eq. (29) is minimized for $\tan^2\theta_\odot = 1$.

With the help of eq. (29) one can study the dependence of the error of $\sin^2\alpha$ on the values of the neutrino mass and mixing parameters and their errors. Figures 1 and 2 show results for a possible determination of a neutrino mass from the tritium spectrum and cosmology, respectively.

As Figs. 1 and 2 demonstrate, establishing under the assumptions made that CP-violation takes place, i.e., that $\sin^2(\alpha_{21}/2) \neq 0, 1$, at 99.73% C.L. requires typically $\tan^2\theta_\odot \gtrsim 0.40$, $m_0^2 \gtrsim 0.70^2$ eV^2, or $\Sigma \gtrsim 1.0$ eV. Further, the value of the CP-violating phase α_{21} should lie approximately within the intervals $\sim (\pi/2 - 3\pi/4)$ and $\sim (5\pi/4 - 3\pi/2)$. The larger the values of $\tan^2\theta_\odot$, and/or m_0^2 or Σ, the larger the interval of values of α_{21} for which CP-violation could be established.

Next we include the effect of the uncertainty in the relevant $(\beta\beta)_{0\nu}$-decay nuclear matrix elements in the analysis by assuming that the value of $|{<\!m\!>}|$ which enters into the expression for $\sigma(\sin^2\alpha)$, eq. (29), is obtained from a measurement of the $(\beta\beta)_{0\nu}$-decay lifetime of given nucleus by using *the maximal allowed value of the nuclear matrix element*, i.e., that $|{<\!m\!>}| = \zeta\Big((|{<\!m\!>}|_{\rm exp})_{\rm MIN}\Big)$, where $\zeta \geq 1$ parametrizes the uncertainty under discussion (see subsection 3.1.1). Figures 1 and 2 correspond then to $\zeta = 1$. In Figures 3 – 6 we exhibit results for $\zeta = 1.5;\ 2.0;\ 3.0$, including in each sub-figure also the results for $\zeta = 1$. As Figs. 3 – 6 show, it would be impossible to make a definite conclusion concerning the CP-violation for values of the parameters considered, $m_0^2 \equiv m_{\bar{\nu}_e}^2 = (0.5^2 - 1.0^2)$ eV^2, $\Sigma = (0.60 - 3.0)$ eV, $\tan^2\theta_\odot = (0.25 - 0.70)$, if $\zeta \gtrsim 3$. Actually, for $\tan^2\theta_\odot \sim 0.25$, $m_0^2 \sim (0.8^2 - 1.0^2)$ eV^2 or $\Sigma \sim (1.5 - 3.0)$ eV, one might even exclude the possibility of $\zeta \gtrsim 3$ at 99.73% C.L. Establishing at 3σ that the phase α_{21} takes a CP-violating value is possible provided $\zeta < 2$, $\tan^2\theta_\odot \gtrsim 0.55$, $m_0^2 \gtrsim 0.70^2$ eV^2, or $\Sigma \gtrsim 1.5$ eV, and if $\alpha_{21} \sim (\pi/2 - 3\pi/4)$ or $\alpha_{21} \sim (5\pi/4 - 3\pi/2)$. The precise sub-intervals of values of α_{21} for which CP-violation could be established depend of the precise values of $\tan^2\theta_\odot$, m_0^2, or Σ. For $\zeta = 2$ this might be done as well, but for a rather limited range of values of α_{21} from the indicated intervals and if $\tan^2\theta_\odot \gtrsim 0.65$, $m_0^2 \gtrsim 0.80^2$ eV^2, or $\Sigma \gtrsim 2.0$ eV. Obtaining an evidence for CP-violation at 2σ level for a given $\zeta \lesssim 2$ is possible for wider ranges of values of α_{21}, $\tan^2\theta_\odot$, m_0^2, or Σ, than those permitting a 3σ "proof".

Similar results are valid for neutrino mass spectrum with inverted hierarchy. In this case the role of m_0^2 is played by $\Delta m^2_{\rm atm}$ (see eq. (7)), which is expected to be measured with a relative error of $\sim 10\%$. Reaching a definite conclusion concerning the CP-violation, as the preceding discussion indicates, requires, in particular, $|{<\!m\!>}|$ to be measured with an error not exceeding $\sim (15-20)\%$.

Once $|{<\!m\!>}|$, $\theta_\odot$ and $m_{\bar{\nu}_e}$ are measured in present and future experiments, constraints at a given C.L. on the allowed values of ζ and $\sin^2\alpha_{21}/2$ can be obtained performing a joined χ^2 analysis of the data on $|{<\!m\!>}|$, $\theta_\odot$ and $m_{\bar{\nu}_e}$.

4 Conclusions

Assuming 3-ν mixing and massive Majorana neutrinos, $(\beta\beta)_{0\nu}$-decay generated only by the (V-A) charged current weak interaction via the exchange of the three Majorana neutrinos, LMA MSW solution of the $\nu_\odot$-problem and neutrino oscillation explanation of the atmospheric neutrino data, we have discussed in the present article the possibility of detecting CP-violation in the lepton sector, associated with Majorana neutrinos, from a measurement of the effective Majorana mass in $(\beta\beta)_{0\nu}$-decay, $|{<\!m\!>}|$. The problem of detection of CP-violation in the lepton sector is one of the most formidable and challenging problems in the studies of neutrino mixing. As was noticed in [28], the measurement of $|{<\!m\!>}|$ alone could exclude the possibility of the two Majorana CP-violating phases α_{21} and α_{31}, present in the lepton mixing matrix in the case of interest, being equal to zero. However, such a measurement cannot rule out without additional input that the two phases take different CP-conserving values. The additional input needed for establishing CP-violation could be, e.g., the measurement of neutrino mass $m_{\bar{\nu}_e}$ in ^{3}H β-decay experiment KATRIN [34], or the cosmological determination of the sum of the three neutrino masses [35], $\Sigma = m_1 + m_2 + m_3$, or a derivation of a sufficiently stringent upper limit on Σ or on the lightest neutrino mass m_1. At present no viable alternative to the measurement of $|{<\!m\!>}|$ for getting information on the Majorana CP-violating phases α_{21} and α_{31} exists, or can be foreseen to exist in the next ~ 8 years. Thus, the present work represents an attempt to determine the conditions under which CP-violation might be detected from a measurement of $|{<\!m\!>}|$ and of $m_{\bar{\nu}_e}$ (or Σ), or of $|{<\!m\!>}|$ and by obtaining a sufficiently stringent upper limit on Σ or m_1.

We have discussed in detail the prospective data on the neutrino mass, mixing and oscillation parameters on which $|{<\!m\!>}|$ depends: $m_{\bar{\nu}_e}$, Σ, $\Delta m^2_{\rm atm}$, $\tan^2\theta_\odot$, $\Delta m^2_\odot$, $|U_{\rm e3}|^2$ or $|U_{\rm e1}|^2$. Considering neutrino mass spectrum with inverted hierarchy and of quasi-degenerate type, we performed a general analysis, the aim of which was to determine what is the maximal uncertainty in the value of $|<m>|$ due to the imprecise knowledge of the corresponding $(\beta\beta)_{0\nu}$-decay nuclear matrix elements, which might still allow one to find CP-violation associated with Majorana neutrinos. This was followed by an example of simplified error analysis of the possibility to establish that the physical Majorana phases take CP-non-conserving values. The analysis is based on prospective input data on $|<m>|$, $m_{\bar{\nu}_e}$, Σ, $\tan^2\theta_\odot$, etc. The effect of the nuclear matrix element uncertainty was included in the analysis. The results thus obtained are illustrated in Figs. 1 – 6.

The possibility of finding the CP-violation of interest requires quite accurate measurements of $|<m>|$ and, say, of $m_{\bar{\nu}_e}$ or Σ, and holds only for a limited range of values of the relevant parameters. More specifically, proving that CP-violation associated with Majorana neutrinos takes place requires, in particular, a relative experimental error on the measured value of $|<m>|$ not bigger than (15 – 20)%, a "theoretical uncertainty" in the value of $|<m>|$ due to an imprecise knowledge of the corresponding nuclear matrix elements smaller than a factor of 2, a value of $\tan^2\theta_\odot \gtrsim 0.55$, and values of the relevant Majorana CP-violating phases (α_{21}, α_{32}) typically within the ranges of $\sim (\pi/2 - 3\pi/4)$ and $\sim (5\pi/4 - 3\pi/2)$.

Acknowledgments. S.T.P. would like to thank H. Ejiri and G. Gratta for useful discussions. The work of W.R. has been supported by the "Bundesministerium für Bildung, Wissenschaft, Forschung und Technologie", Bonn under contract No. 05HT1PEA9. The work of S.T.P. was supported in part by the EC network HPRN-CT-2000-00152.

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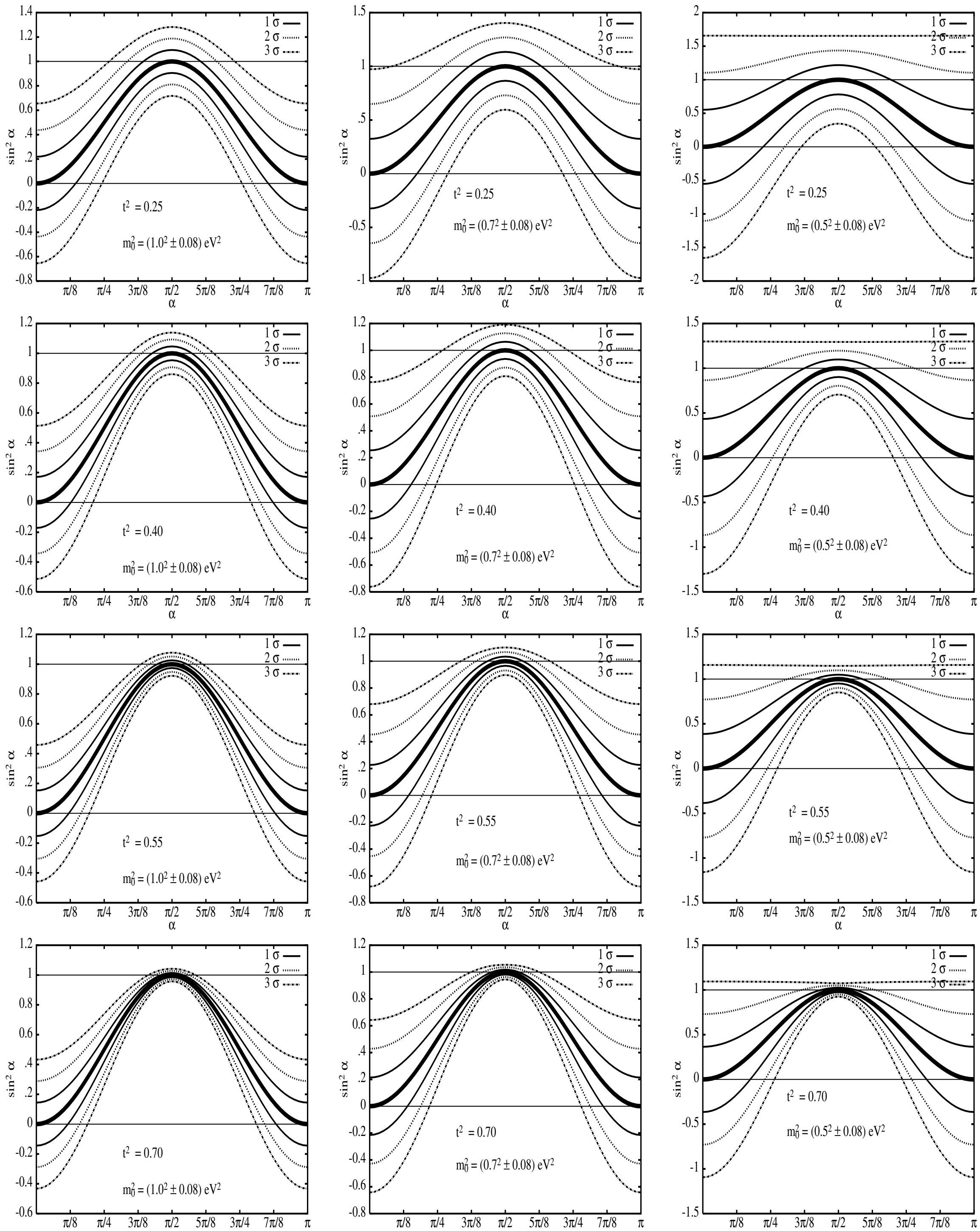

Figure 1: The error on $\sin^2\alpha$ as a function of $\alpha \equiv \alpha_{21}/2$ in the case of QD neutrino mass spectrum (see eq. (27)) for different values of $m_0^2 \equiv m_{\bar{\nu}_e}^{\;2}$ and $t^2 \equiv \tan^2\theta_\odot$. The neutrino mass parameter $m_0^2 \equiv m_{\bar{\nu}_e}^{\;2} \equiv m_{1,2,3}^2$ is assumed to be determined in the ^{3}H β-decay experiment KATRIN [34] with an error of 0.08 eV^2. The error on $\tan^2\theta_\odot$ is 5%, while that on $|<m>|$ is given by eq. (28). The 1 σ range is within the solid lines, the 2σ (3σ) error band is within the dashed (dash-dotted) lines.

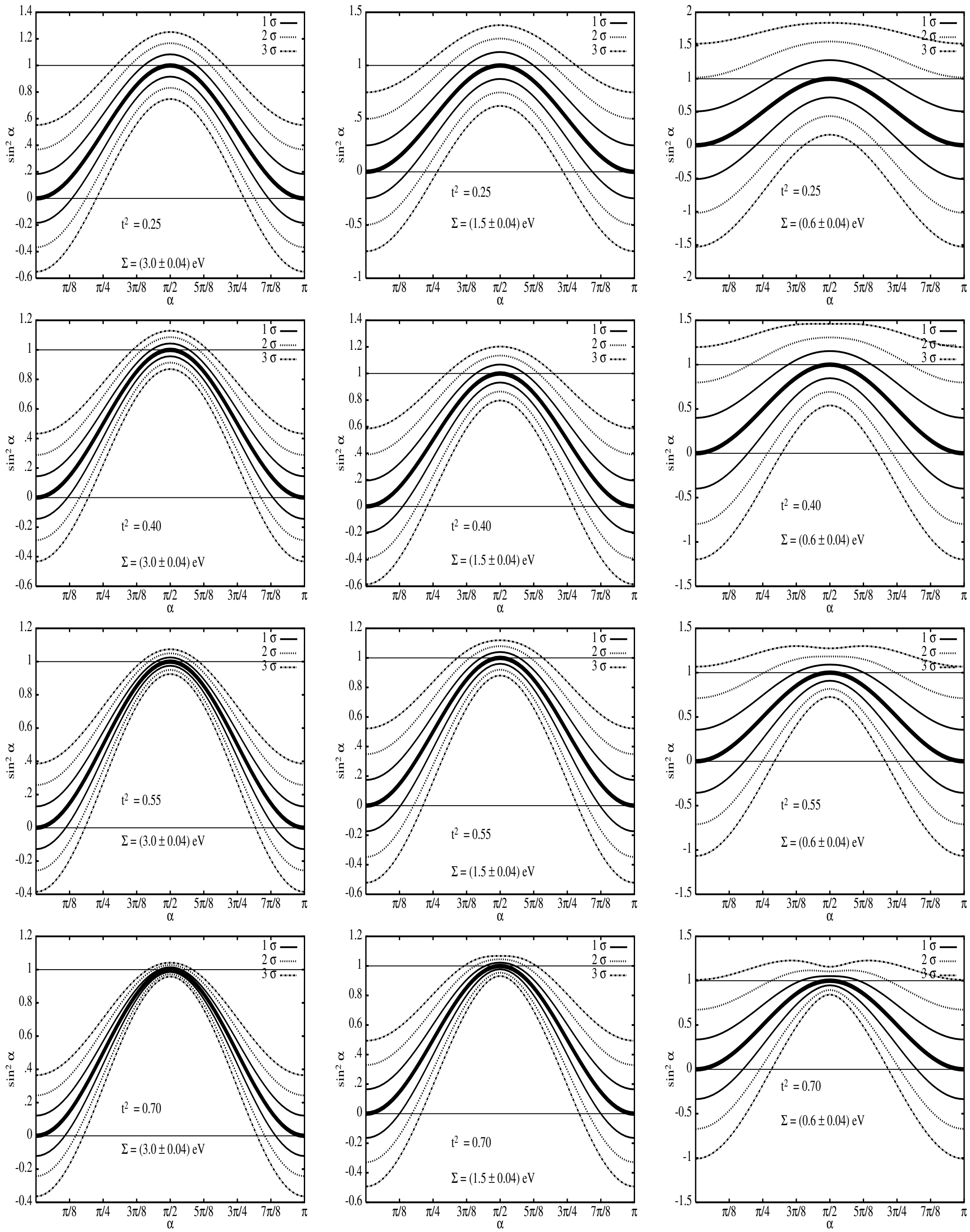

Figure 2: The error on $\sin^2\alpha$ as a function of $\alpha \equiv \alpha_{21}/2$ in the case of QD neutrino mass spectrum (see eq. (27)) for different values of $\Sigma = 3m_0$ ($m_0 \equiv m_{\bar{\nu}_e}$) and $t^2 \equiv \tan^2\theta_\odot$. The sum of the neutrino masses Σ is assumed to be determined from astrophysical and cosmological data with an error of 0.04 eV [35]. The error on $\tan^2\theta_\odot$ is 5%, while the error on $|\langle m\rangle|$ is given by eq. (28). The 1 σ range is within the solid lines, the 2σ (3σ) error band is within the dashed (dash-dotted) lines.

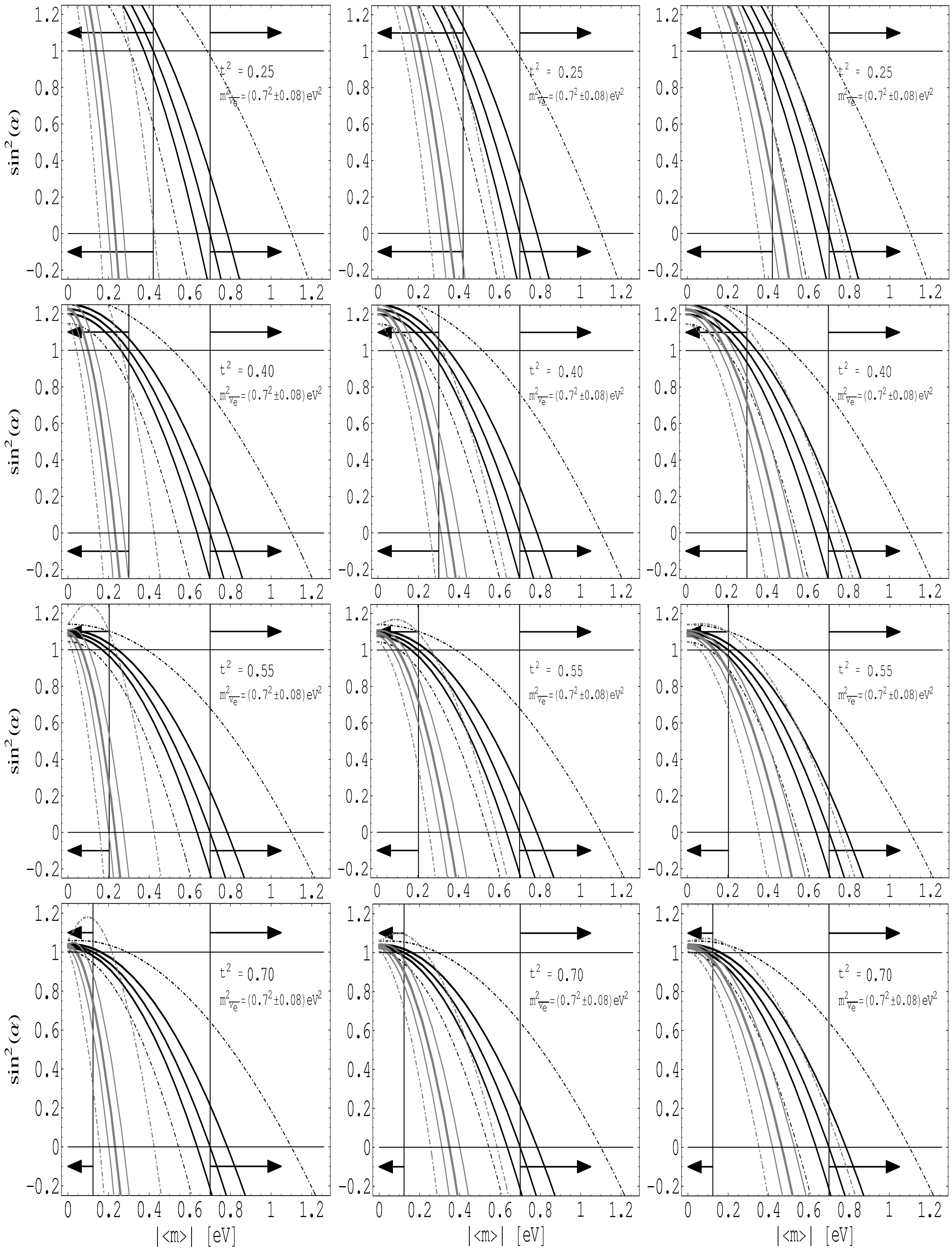

Figure 3: The same as in Fig. 1 for $m_0^2 = m_{\bar{\nu}_e}{}^2 = (0.70^2 \pm 0.08)$ eV^2 and three values of nuclear matrix element uncertainty factor ζ: 3.0 (left panels), 2.0 (middle panels) and 1.5 (right panels). The 1 σ range shown for $\zeta = 1.5;\ 2.0;\ 3.0$ ($\zeta = 1.0$) is within the light-gray (black) solid lines, the 3σ error band is within the light-gray (black) dash-dotted lines. The arrows denote the values of $|{<}m{>}|$ which lie outside the allowed region in the case of the QD neutrino mass spectrum.

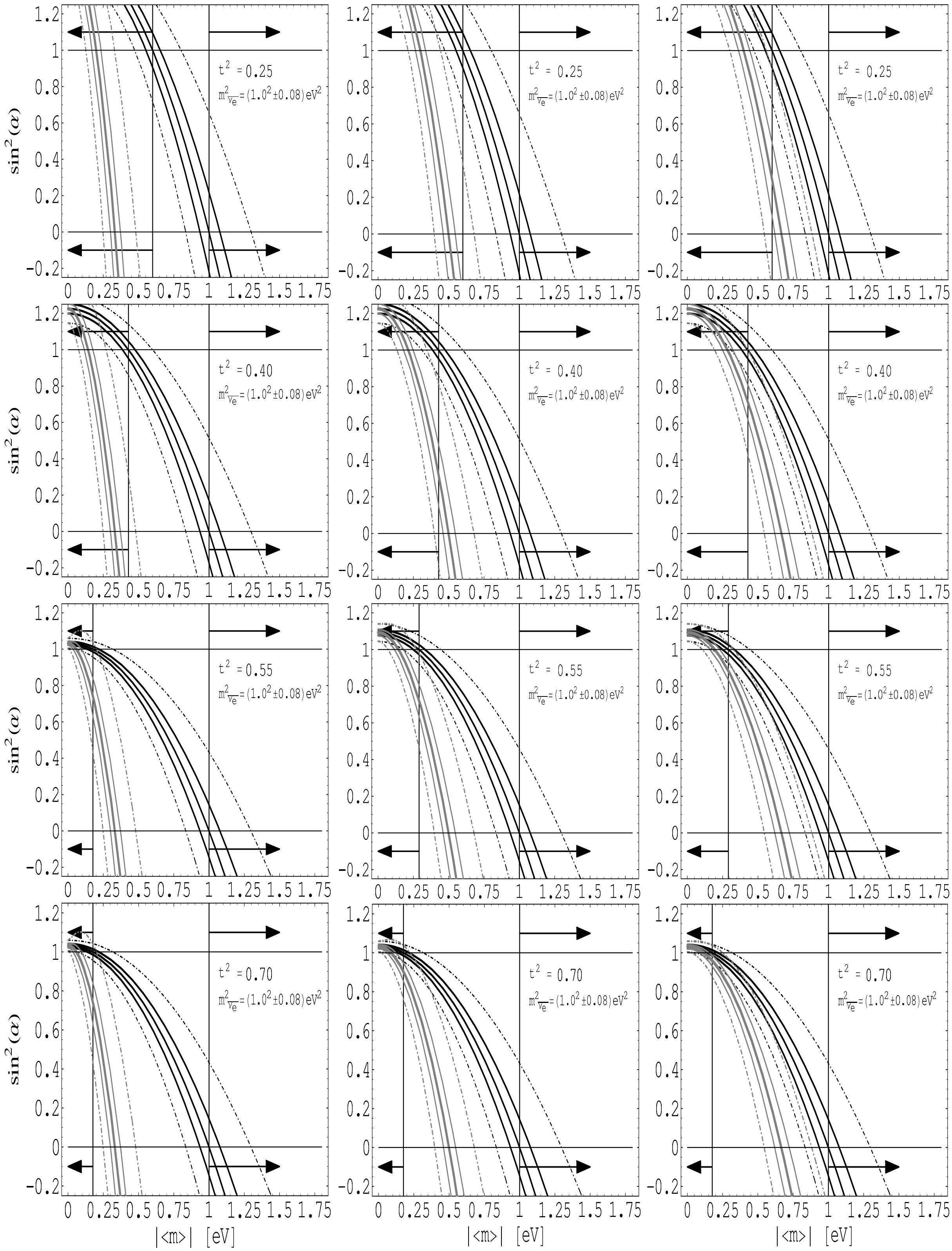

Figure 4: The same as in Fig. 1 for $m_0^2 = m_{\bar{\nu}_e}{}^2 = (1.0^2 \pm 0.08)$ eV^2 and three values of nuclear matrix element uncertainty factor ζ: 3.0 (left panels), 2.0 (middle panels) and 1.5 (right panels). The 1 σ range shown for $\zeta = 1.5;\ 2.0;\ 3.0$ ($\zeta = 1.0$) is within the light-gray (black) solid lines, the 3σ error band is within the light-gray (black) dash-dotted lines. The arrows denote the values of $|<m>|$ which lie outside the allowed region in the case of the QD neutrino mass spectrum.

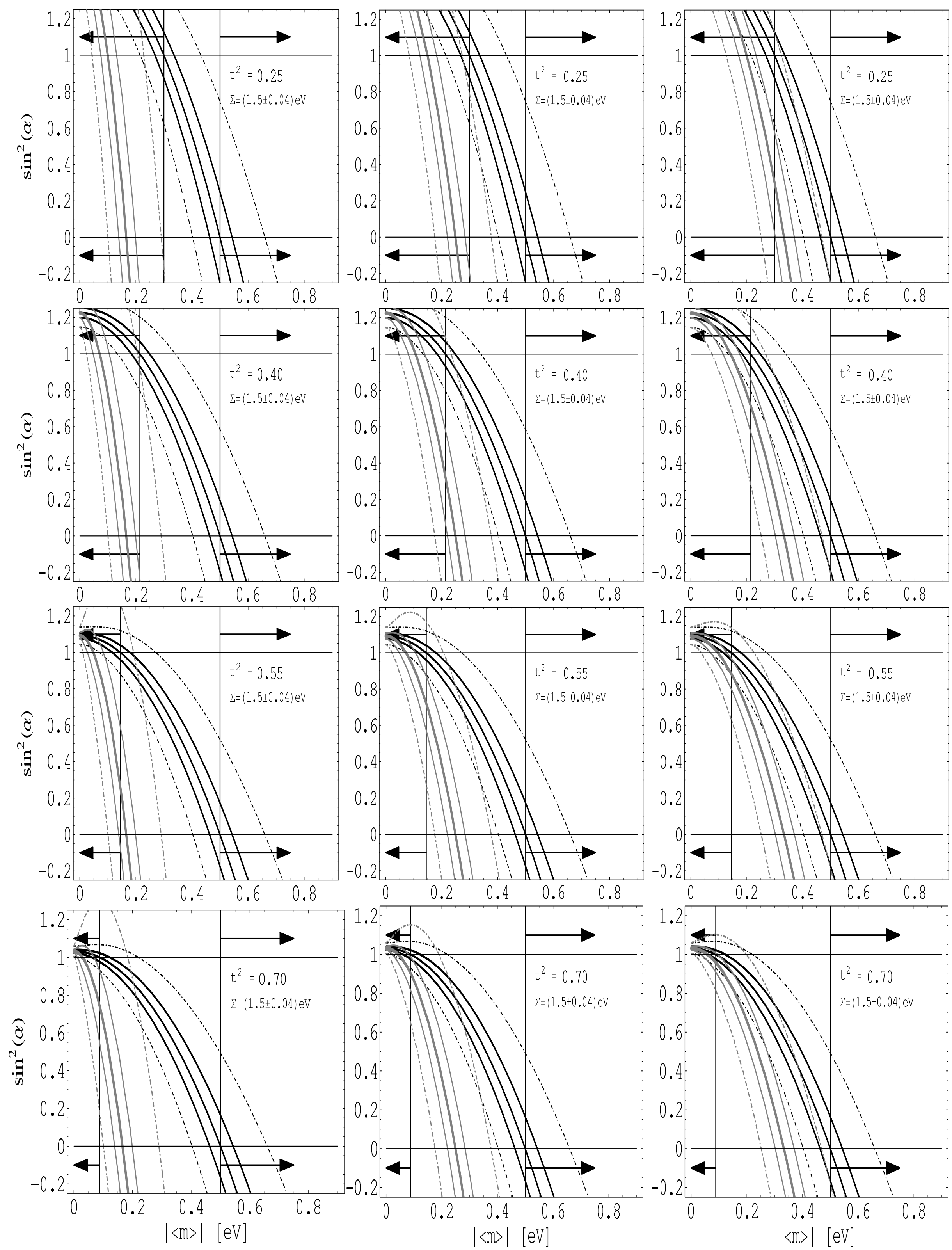

Figure 5: The same as in Fig. 2 for $\Sigma = (1.5 \pm 0.04)$ eV and three values of nuclear matrix element uncertainty factor ζ: 3.0 (left panels), 2.0 (middle panels) and 1.5 (right panels). The 1 σ range shown for $\zeta = 1.5$; 2.0; 3.0 ($\zeta = 1.0$) is within the light-gray (black) solid lines, the 3σ error band is within the light-gray (black) dash-dotted lines. The arrows denote the values of $|{<}m{>}|$ which lie outside the allowed region in the case of the QD neutrino mass spectrum.

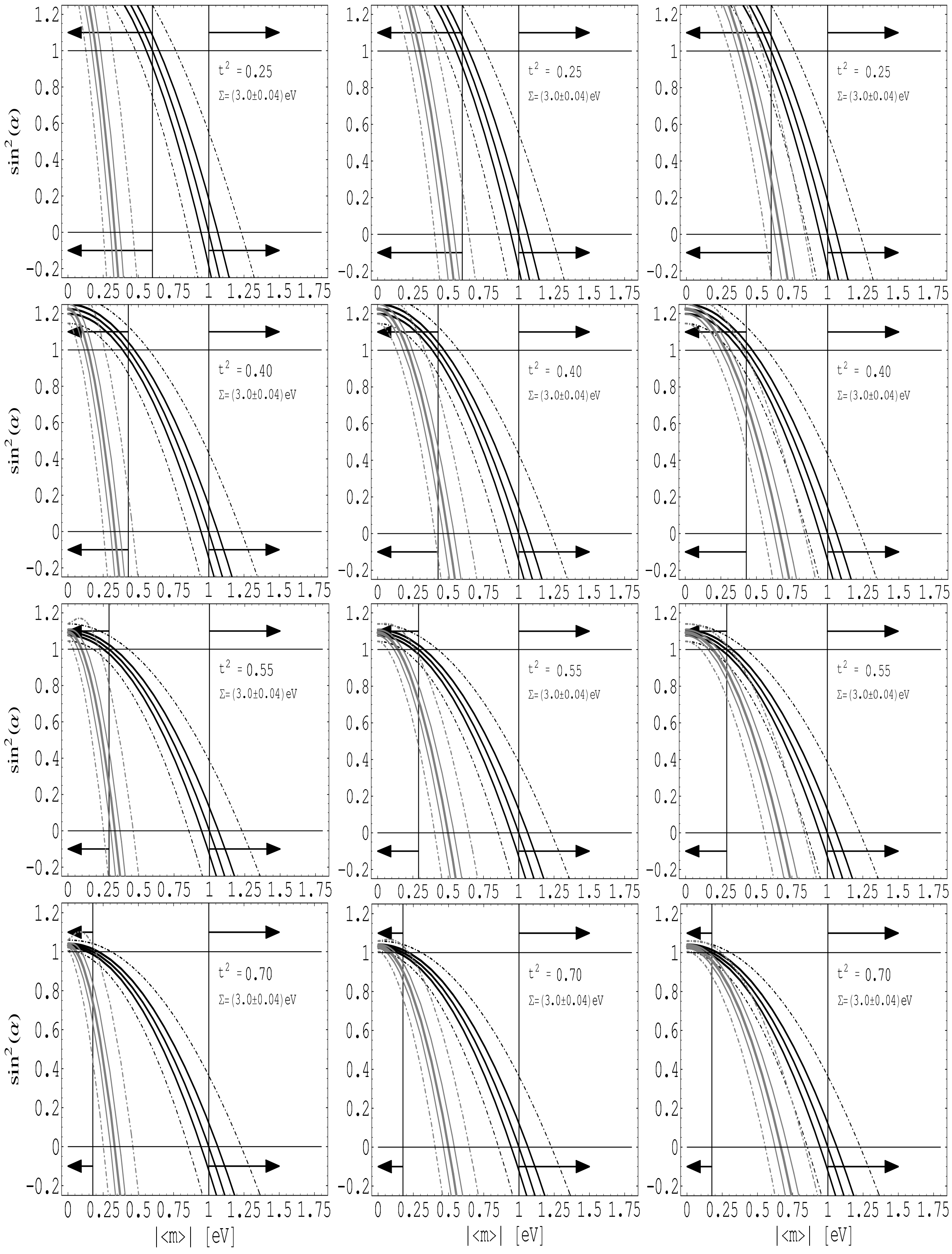

Figure 6: The same as in Fig. 2 for $\Sigma = (3.0 \pm 0.04)$ eV and three values of nuclear matrix element uncertainty factor ζ: 3.0 (left panels), 2.0 (middle panels) and 1.5 (right panels). The 1 σ range shown for $\zeta = 1.5$; 2.0; 3.0 ($\zeta = 1.0$) is within the light-gray (black) solid lines, the 3σ error band is within the light-gray (black) dash-dotted lines. The arrows denote the values of $|<m>|$ which lie outside the allowed region in the case of the QD neutrino mass spectrum.